Warm-up (15 min)

1. Attendance: https://forms.gle/iMXPHXs6xW4qn5Ht8

2. Enroll in CS 103ACE at https://simpleenroll.stanford.edu using your enrollment code (come to me if you do not have one)

3. React to my message in the main slack channel

4. Midterm 1 review session time form: https://forms.gle/9K9q1aq8JUKmLRhh7

5. Work on problems 1 & 2 with your table!

Connective	Meaning	Negation(s)
A ∧ B	A and B	
A ∨ B	A or B	
¬A	Not A	
A → B	Implication: if A, then B	
A ↔ B	Biconditional: if A then B, and if B then A	
⊤	True	
⊥	False	

Connective	Meaning	Negation(s)
$A \wedge B$	A and B	$\neg A \vee \neg B$ $A \rightarrow \neg B$
$A \vee B$	A or B	$\neg A \wedge \neg B$
$\neg A$	Not A	A
$A \rightarrow B$	Implication: if A, then B	$A \wedge \neg B$
$A \leftrightarrow B$	Biconditional: if A then B, and if B then A	$A \leftrightarrow \neg B$ $\neg A \leftrightarrow B$
$\top$	True	$\bot$
$\bot$	False	$\top$

Expression	English Translation	Negation of Expression
$\forall x.(P(x))$		
$\exists x.(P(x))$		
$\forall x.(\neg P(x))$		
$\exists x.(\neg P(x))$		
$\forall x.(A(x) \rightarrow B(x))$		
$\exists x.(A(x) \wedge B(x))$		
$\forall x.(A(x) \rightarrow \neg B(x))$		
$\exists x.(A(x) \wedge \neg B(x))$		

Expression	English Translation	Negation of Expression
$\forall x.(P(x))$	Everything is a P.	$\exists x.(\neg P(x))$
$\exists x.(P(x))$	Something is a P.	$\forall x.(\neg P(x))$
$\forall x.(\neg P(x))$	Nothing is a P.	$\exists x.(P(x))$
$\exists x.(\neg P(x))$	Something isn't a P.	$\forall x.(P(x))$
$\forall x.(A(x) \rightarrow B(x))$	All A's are B's.	$\exists x.(A(x) \wedge \neg B(x))$
$\exists x.(A(x) \wedge B(x))$	Some A is a B.	$\forall x.(A(x) \rightarrow \neg B(x))$
$\forall x.(A(x) \rightarrow \neg B(x))$	No A is a B.	$\exists x.(A(x) \wedge B(x))$
$\exists x.(A(x) \wedge \neg B(x))$	Some A isn't a B.	$\forall x.(A(x) \rightarrow B(x))$

Ice-Breaker

/ help Elena learn everyone's names

1. Name
2. Pronouns
3. Something you did this weekend (that was not school related)

First-Order Logic Tips

"All *A*s are *B*s"

$$\forall x. (A(x) \rightarrow B(x))$$

"Some *A*s are *B*s"

$$\exists x. (A(x) \wedge B(x))$$

- To evaluate "for all x…", try substituting every possible object in for "x"
  - If the inner statement is ever false, the entire statement is false
  - Always true in an empty world
- To evaluate "there exists x…", , try to find just one object that you can substitute in for "x"
  - If the inner statement is ever true, the entire statement is true
  - Always false in an empty world

	Meaning	Negation
$\forall x.\ P(x)$		
$\exists x.\ P(x)$		
$\forall x.\ \neg P(x)$		
$\exists x.\ \neg P(x)$		

	When is this true?	When is this false?
$\forall x.\ P(x)$	For all objects x, $P(x)$ is true.	$\exists x.\ \neg P(x)$
$\exists x.\ P(x)$	There is an x where $P(x)$ is true.	$\forall x.\ \neg P(x)$
$\forall x.\ \neg P(x)$	For all objects x, $P(x)$ is false.	$\exists x.\ P(x)$
$\exists x.\ \neg P(x)$	There is an x where $P(x)$ is false.	$\forall x.\ P(x)$

$$\forall x. \ (A(x) \rightarrow B(x)) \qquad \exists x. \ (A(x) \land B(x))$$

$$\forall x. \ (A(x) \rightarrow \neg B(x)) \qquad \exists x. \ (A(x) \land \neg B(x))$$

"All As are Bs"

$$\forall x. \, (A(x) \rightarrow B(x))$$

"Some As are Bs"

$$\exists x. \, (A(x) \wedge B(x))$$

"No As are Bs"

$$\forall x. \, (A(x) \rightarrow \neg B(x))$$

"Some As aren't Bs"

$$\exists x. \, (A(x) \wedge \neg B(x))$$

Problem 3: Interpreting nested statements

Translating:

- Work outside in - one step at a time
- Every time you see a quantifier, replace it:
  - $\exists$ x. = "there is an x where…"
  - $\forall$ y. = "for every y…"
- Simplify patterns based on the 4 basic forms

Checking if a statement is true:

- Assign specific examples
  - $\exists$ => inner statement must be true for <u>some</u> choice
  - $\forall$ => inner statement must be true for <u>every</u> choice

Problem 4: Translating English to Logic

- Keep asking yourself: Which basic form applies?
  - Can you turn it into a statement about
    everything in some category (universally quantified)
    or some things from the category (existentially quantified)?
- One step at a time
- Give names to any entities mentioned in the statement

Special cases: "exactly one", "two different"

Problem 5: Negations

Negations are important for indirect proofs involving first-order definitions

- "Pushing the negation inward" demo
- Replace complicated expressions with symbols
- One step at a time
- To check your work: use truth table tool

Our proof-writing toolkit, now with logic

- To show $\forall x. P(x)$...
  - Have the reader **pick an arbitrary** x.
  - Then, show that P(x) is true.
- To show $\exists x. Q(x)$...
  - You **give a value** for x.
  - Then, show that Q(x) is true for that value of x.
- To show $A(x) \rightarrow B(x)$...
  - **Assume** that A(x) is true, **Prove** that B(x) is true.
  - Or contrapositive: **Assume** ¬B(x), **Prove** ¬A(x).